

5-9

Study Guide and Intervention

Complex Numbers

Add and Subtract Complex Numbers

Complex Number	A complex number is any number that can be written in the form $a + bi$, where a and b are real numbers and i is the imaginary unit ($i^2 = -1$). a is called the real part, and b is called the imaginary part.
Addition and Subtraction of Complex Numbers	Combine like terms. $(a + bi) + (c + di) = (a + c) + (b + d)i$ $(a + bi) - (c + di) = (a - c) + (b - d)i$

Example 1Simplify $(6 + i) + (4 - 5i)$.

$$\begin{aligned}
 (6 + i) + (4 - 5i) \\
 &= (6 + 4) + (1 - 5)i \\
 &= 10 - 4i
 \end{aligned}$$

Example 2Simplify $(8 + 3i) - (6 - 2i)$.

$$\begin{aligned}
 (8 + 3i) - (6 - 2i) \\
 &= (8 - 6) + [3 - (-2)]i \\
 &= 2 + 5i
 \end{aligned}$$

To solve a quadratic equation that does not have real solutions, you can use the fact that $i^2 = -1$ to find complex solutions.

Example 3Solve $2x^2 + 24 = 0$.

$2x^2 + 24 = 0$	Original equation
$2x^2 = -24$	Subtract 24 from each side.
$x^2 = -12$	Divide each side by 2.
$x = \pm\sqrt{-12}$	Take the square root of each side.
$x = \pm 2i\sqrt{3}$	$\sqrt{-12} = \sqrt{4} \cdot \sqrt{-1} \cdot \sqrt{3}$

Exercises

Simplify.

1. $(-4 + 2i) + (6 - 3i)$

2. $(5 - i) - (3 - 2i)$

3. $(6 - 3i) + (4 - 2i)$

4. $(-11 + 4i) - (1 - 5i)$

5. $(8 + 4i) + (8 - 4i)$

6. $(5 + 2i) - (-6 - 3i)$

7. $(12 - 5i) - (4 + 3i)$

8. $(9 + 2i) + (-2 + 5i)$

9. $(15 - 12i) + (11 - 13i)$

10. i^4

11. i^6

12. i^{15}

Solve each equation.

13. $5x^2 + 45 = 0$

14. $4x^2 + 24 = 0$

15. $-9x^2 = 9$

5-9 Study Guide and Intervention (continued)**Complex Numbers****Multiply and Divide Complex Numbers****Multiplication of Complex Numbers**

Use the definition of i^2 and the FOIL method:
 $(a + bi)(c + di) = (ac - bd) + (ad + bc)i$

To divide by a complex number, first multiply the dividend and divisor by the **complex conjugate** of the divisor.

Complex Conjugate

$a + bi$ and $a - bi$ are complex conjugates. The product of complex conjugates is always a real number.

Example 1 Simplify $(2 - 5i) \cdot (-4 + 2i)$.

$$\begin{aligned}
 (2 - 5i) \cdot (-4 + 2i) &= 2(-4) + 2(2i) + (-5i)(-4) + (-5i)(2i) && \text{FOIL} \\
 &= -8 + 4i + 20i - 10i^2 && \text{Multiply.} \\
 &= -8 + 24i - 10(-1) && \text{Simplify.} \\
 &= 2 + 24i && \text{Standard form}
 \end{aligned}$$

Example 2 Simplify $\frac{3 - i}{2 + 3i}$.

$$\begin{aligned}
 \frac{3 - i}{2 + 3i} &= \frac{3 - i}{2 + 3i} \cdot \frac{2 - 3i}{2 - 3i} && \text{Use the complex conjugate of the divisor.} \\
 &= \frac{6 - 9i - 2i + 3i^2}{4 - 9i^2} && \text{Multiply.} \\
 &= \frac{3 - 11i}{13} && i^2 = -1 \\
 &= \frac{3}{13} - \frac{11i}{13} && \text{Standard form}
 \end{aligned}$$

Exercises

Simplify.

1. $(2 + i)(3 - i)$
2. $(5 - 2i)(4 - i)$
3. $(4 - 2i)(1 - 2i)$
4. $(4 - 6i)(2 + 3i)$
5. $(2 + i)(5 - i)$
6. $(5 - 3i)(-1 - i)$
7. $(1 - i)(2 + 2i)(3 - 3i)$
8. $(4 - i)(3 - 2i)(2 + i)$
9. $(5 - 2i)(1 - i)(3 + i)$
10. $\frac{5}{3 + i}$
11. $\frac{7 - 13i}{2i}$
12. $\frac{6 - 5i}{3i}$
13. $\frac{4 - 2i}{3 + i}$
14. $\frac{-5 - 3i}{2 - 2i}$
15. $\frac{3 + 4i}{4 - 5i}$
16. $\frac{3 + i\sqrt{5}}{3 - i\sqrt{5}}$
17. $\frac{4 - i\sqrt{2}}{i\sqrt{2}}$
18. $\frac{\sqrt{6} + i\sqrt{3}}{\sqrt{2} - i}$